Find (using IBP)

(1) $\int xe^{-x}\,dx$

(2) $\int \frac{x^3}{(1+x^2)^2}\,dx$

(3) $\int \arctan x\,dx$

(4) $\int \frac{\ln x}{x^2}\,dx$

31|8|25

Find (using IBP)

(1) $\int xe^{-x}\,dx$

(2) $\int \frac{x^3}{(1+x^2)^2}\,dx$

(3) $\int \arctan x\,dx$

(4) $\int \frac{\ln x}{x^2}\,dx$

(1) $\int xe^{-x}\,dx$ $\quad x = u$, $e^{-x} = v'$

$= -e^{-x}x - \int -e^{-x}\cdot 1\,dx$

$= -xe^{-x} + \frac{e^{-x}}{-1} + C$

$= -e^{-x}(1+x) + C$

(2) $\int \frac{x^3}{(1+x^2)^2}\,dx$ $\quad u = x^2 \Rightarrow du = 2x\,dx$

$= \int \frac{u}{(1+u)^2}\,\frac{1}{2}\,du$

$= \frac{1}{2}\left(u\,\frac{1}{-(1+u)} - \int \frac{1}{-(1+u)}\cdot(1)\,du\right)$

$= -\frac{1}{2}\left(\frac{u}{1+u} - \ln|1+u| + C_1\right)$

$= -\frac{x^2}{2(1+x^2)} + \frac{\ln|1+x^2|}{2} + C$

(3) $\int \arctan x\,dx$ $\quad u = \arctan x \Rightarrow \tan u = x$, $dx = \sec^2 u\,du$

$= \int u\cdot\sec^2 u\,du$

$= u\tan u + \int 1\cdot\tan u\,du$

$= u\tan u + \ln|\cos u| + C$

$= \arctan x\cdot x + \ln\left|\frac{1}{\sqrt{1+x^2}}\right| + C$

(4) $\int \frac{\ln x}{x^2}\,dx = \ln x\cdot\left(-\frac{1}{x}\right) - \int \frac{1}{x}\cdot\left(\frac{-1}{x}\right)dx$

$\ln x = u$ $\quad = -\frac{\ln x}{x} + \left(-\frac{1}{x}\right) + C$

$\frac{1}{x^2} = v'$ $\quad = -\frac{1}{x}(\ln x + 1) + C$